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RESULTS OF AEROSAT
CHANNEL SIMULATION TESTS
Q-M/PSK VOICE/DATA MODEM
TSC RANGING MODEM

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#### **PREFACE**

The Transportation Systems Center of the U.S. Department of Transportation has established a modem evaluation and channel simulation facility for the purpose of evaluating candidate modems for Aeronuatical Satellite (AEROSAT). The AEROSAT channel is characterized by a number of parameters which can vary according to sea state, aircraft-to-satellite elevation angle, and aircraft velocity. These parameters are electronically simulated by the facility's channel simulator. Thus, a modem's performance can be evaluated under real operational conditions without actual field testing which is costly and time consuming. Some results of testing two modems, a hybrid modem (voice plus data on a single carrier), and the TSC developed ranging modem, are presented.

The author wishes to thank the following for their contributions: Joseph Gutwein, Peter Engels, Robert Bland, Robert Wisleder, and Peter Mauro.

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# TABLE OF CONTENTS

Section		Page
1.	INTRODUCTION	1
2.	AEROSAT CHANNEL SIMULATOR	2
3.	Q-M/PSK HYBRID MODEM	4
4.	RANGING MODEM	6
5.	RESULTS - Q-M/PSK HYBRID MODEM	9
6.	RESULTS - RANGING MODEM	12
7.	CONCLUSIONS	18
	APPENDIX - CALCULATION OF RMS PHASE JITTER	19
	REFERENCES	21
	LIST OF FIGURES	
Figure		Page
1	AERONAUTICAL SATELLITE CHANNEL SIMULATOR	3
2	RANGING MODEM LABORATORY EVALUATION TEST SETUP	7
3	PERFORMANCE OF Q-M/PSK HYBRID MODEM IN THE VOICE PLUS DATA MODE	10
4	RANGING MODEM PERFORMANCE, RMS PHASE JITTER, $\sigma_{rms}$ , VS CARRIER POWER-TO-NOISE POWER	
	DENSITY, C/N <sub>o</sub>	13
5	RANGING MODEM PERFORMANCE, RMS PHASE JITTER,	
	DENSITY, C/N <sub>o</sub> , WITH MULTIPATH (CODES 1 AND 2)	15
6	RANGING MODEM PERFORMANCE, RMS PHASE JITTER, $\sigma_{rms}$ , VS CARRIER POWER-TO-NOISE POWER DENSITY,	13
	C/N <sub>o</sub> , WITH MULTIPATH (CODES 3 AND 4)	16
7	COMPARISON OF PERFORMANCE OF CODES 3 AND 4 TO THEORETICAL (RMS PHASE JITTER, $\sigma_{rms}$ , VS CARRIER POWER-TO-NOISE POWER	
	DENSITY, C/N <sub>O</sub> )	17

#### LIST OF ABBREVIATIONS AND SYMBOLS

AEROSAT Aeronautical Satellite

ATS-6 Applications Technology Satellite 6

BER Bit error rate

 $BW_{m}$  Multipath fading bandwidth

C/M Carrier-to-multipath ratio

C/N Carrier power-to-noise power density

DECPSK Differentially encoded phase-shift keying

f<sub>D</sub> Doppler frequency offset

f<sub>RD</sub> Relative doppler frequency offset

PB Phonetically balanced

PN Pseudo random noise

PSK Phase-shift keyed

Q-M/PSK Quadrature modulation/phase-shift keyed

rms Root-mean-square

rv Random variable

 ${\sf SNR}_{\sf I}$  Loop signal-to-noise ratio

S/N Signal-to-additive noise ratio

TSC Transportation Systems Center

T<sub>D</sub> Multipath group delay

ΔS Differential distance

ΔT Phase difference

 $\Delta \tau$  Differential time

Θ<sup>2</sup>n Average loop phase jitter

σ Phase jitter

 $\sigma_{rms}$  Rms phase jitter

v Average velocity

## 1. INTRODUCTION

The Transportation Systems Center (TSC) has developed a system for the laboratory evaluation of candidate modems for the Aeronautical Satellite (AEROSAT) Avionics (ref. 1). The communication channel, satellite to aircraft, is characterized by a number of parameters, some of which are signal-to-additive noise ratio (S/N), carrier-to-multipath ratio (C/M), multipath fading bandwidth  $(BW_{M})$ , multipath group delay  $(T_{n})$ , doppler frequency offset  $(f_{n})$ , and relative doppler frequency offset  $(f_{RD})$ . With the AEROSAT channel simulation facility, the TSC has been evaluating candidate voice, data, and ranging modems for the AEROSAT Avionics (ref. 2). This report summarizes some of the results obtained from July 1, 1975 to January 1, 1976. Tested during this period was the Quadrature-Modulation/Phase-Shift Keyed (Q-M/PSK) Hybrid Modem (ref. 3) in the voice plus data mode for bit error rate (BER) performance as a function of carrier power-to-noise power density  $(C/N_0)$  under various multipath conditions. Also tested was the TSC Ranging Modem (ref. 4); measured was rms phase jitter as a function of  $\mathrm{C/N}_{\mathrm{O}}$  and various multipath conditions as parameters. Both of these modems were used during the Applications Technology Satellite 6 (ATS-6) tests performed from September 1974 to April 1975.

### 2. AEROSAT CHANNEL SIMULATOR

The AEROSAT Channel Simulator (ref. 1) simulates the communication channel between satellite and mobile, either ship or aircraft. See Figure 1. The satellite communication channel is characterized by a number of parameters which can vary according to sea conditions and the geometry that can exist between the aircraft and satellite. The Channel Simulator accounts for these effects by electronically simulating multipath, multipath delay, doppler, and relative doppler. The carrier-to-multipath ratio (C/M) can be varied to create the effects of light, medium, or heavy multipath fading (for example, typical values might be C/M = 11 dB or greater for light fading, C/M = 8 dB for medium fading, and C/M = 5 dB for heavy fading. Another parameter controlled is the multipath fading bandwidth. The energy of the multipath may be concentrated in any bandwidth from 10 Hz to 2 kHz. There is also the ability to set up a multipath "profile". To create the multipath, a tapped delay line is used with five taps 2 µs apart. The output of each tap is then mixed with noise in a complex fashion to produce multipath. The power output and bandwidth of each is controllable. All the multipath "profiles" are then combined to produce the final multipath configuration. The simulator can offset the output frequency from the input to simulate doppler effects and also offset the multipath frequency to simulate relative doppler. The time delay between the direct path and multipath is also variable (3 time delays are available: 5, 30, and 55  $\mu s$ ). At the output of the simulator the whole spectrum is added to Gaussian white noise to simulate receiver front end noise.

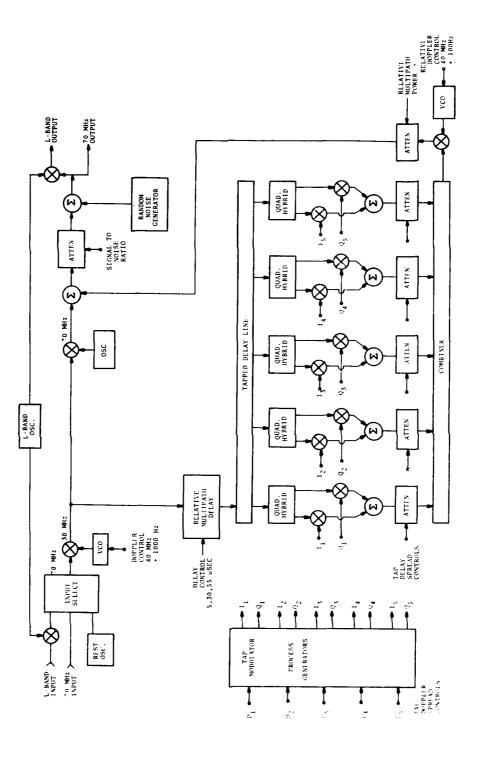


FIGURE 1. AERONAUTICAL SATELLITE CHANNEL SIMULATOR

## 3. Q-M/PSK HYBRID MODEM

The Q-M/PSK Hybrid Modem was developed by Bell Aerospace of Buffalo, New York. It has the capability for transmission and reception of simultaneous voice and data signals on a single constant envelope carrier and occupying the same bandwidth. The modem has three modes of operation: voice, data, and voice plus data (ref. 3). The results presented herein are for the quadrature voice plus data mode.

In the voice mode, the voice signal is double-sideband suppressed carrier modulated onto the 90° component of the carrier. This is then summed with a residual 0° component of the carrier. The combined signal is then passed through a bandpass limiter to generate a constant amplitude signal.

In the data mode, the digital data consisting of a pseudorandom noise sequence is differentially encoded and then phase-shift keyed onto the 70 MHz carrier. The bit rate is either 1,200 or 2,400 bits per second, and may be either internally or externally generated. Internal generation provides for pseudorandom noise (PN) codes of bit length  $2^6$  -1 = 63,  $2^7$  -1 =127, or  $2^{11}$  -1 = 2047. After detection in the demodulator, a comparison of the received PN signal is made with the transmitted sequence from which errors are observed.

In the voice-plus-data mode, the differentially encoded phase-shift keyed (PSK) data is modulated onto the in-phase (0°) carrier while the voice signal is modulated onto the quadrature (90°) carrier component. The combined signal is bandpass limited. During normal operation, the peak modulation angle is set at 58°

which provides a root-mean-square modulation of 40° so that the power is split almost evenly between the voice and data signals. During voice pauses, all the signal power is concentrated in the data signal.

The 70 MHz signal is received and down-converted to 455 kHz. The 455 kHz intermediate frequency signal is coherently demodulated in conjuction with the automatic gain control circuitry to produce separate voice and data signals. The data signal is optimally filtered and detected by the integrate and dump network. There is a clock synchronization network to provide synchronization between the received data and the lock clock. The data is then differentially decoded and fed to the code synchronization network to be compared with a locally generated code for bit error detection. The voice signal is demodulated and fed to headphone and recorder outputs. The carrier power-to-noise power density  $(\text{C/N}_0)$  measurement network converts the received signal-plusnoise into a voltage level for driving the  $\text{C/N}_0$  meter and for tape recording and monitoring.

### 4. RANGING MODEM

The TSC developed ranging modem incorporates the Binor code in a side tone ranging system. The Binor code is a code generated from the majority logic of a discrete series of tones, each tone half the frequency of the preceding tone (ref. 4). Four of eight possible codes were used during the ATS-6 technology tests; thus, it is these four codes for which results are presented here. Code 1 is a narrowband code, clock tone of 19 kHz, which distributes the code power evenly among all tones. Code 3 is also a narrowband code, clock tone of 19 kHZ, which has twice as much power in the clock tone compared to each other tone. Code 2 is a wideband code, clock tone of 156 kHz, and code 4 is also a wideband code, clock tone of 156 kHz, which like code 3 has twice as much power in the clock tone as in each other tone.

The block diagram of the test setup used to evaluate the ranging modem in the laboratory is shown in Figure 2. The output of the ranging modulator at 70 MHz (0 dBm) goes to the channel simulator and then through the attenuator ATT $_{\rm C}$ . This attenuator controls the level of the signal going to the demodulator. The noise generator, which puts out gaussian white noise from 0 - 20 MHz, is mixed with the 60 MHz oscillator to provide noise at 70 MHz (BW = 20 MHz). This noise simulates the receiver front end noise encountered in the channel. In future systems the noise generator and oscillator will be replaced by a noise module which will provide noise at 70 MHz. An amplifier will follow which will provide the necessary level to establish C/No values in the right range (i.e., 36 dB Hz and greater). The noise attenuator ATT $_{\rm M}$ 

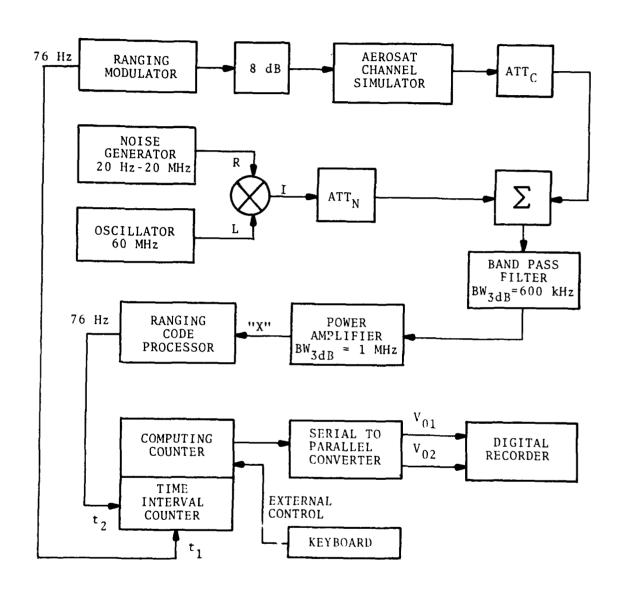


FIGURE 2. RANGING MODEM LABORATORY EVALUATION TEST SETUP

controls the level of the noise signal to the demodulator to establish C/No. To measure C/No,  ${\rm ATT}_{\rm C}$  is set to maximum attenuation and the noise power measured at "X" with  ${\rm ATT}_{\rm N}$  = 0. The  ${\rm ATT}_{\rm N}$  is set to maximum attenuation and the signal power measured at "X" with  ${\rm ATT}_{\rm C}$  = 0. Thus, with the signal power known, noise power known, and by using the bandpass filter at 70 MHz (BW = 600 kHz) the C/No is easily calculated as [P<sub>S</sub> -P<sub>N</sub> + 58] dB-Hz. Setting  ${\rm ATT}_{\rm C}$  to the correct value for the input power range of the demodulator,  ${\rm ATT}_{\rm N}$  can be varied to give the desired C/No value. The computing counter uses the reference 76 Hz from the modulator along with the demodulated 76 Hz to measure the range jitter. The serial to parallel converter is used to drive the digital recorder.

The output of the ranging code processor is the 76 Hz square wave. The phase difference ( $\Delta T$ ) which exists between the transmitted and received 76 Hz tone is the relative range. This  $\Delta T$  can vary due to receiver noise, loop noise in the phase lock loop of the demodulator, and quantization error, since this is a digital system. All the variations in  $\Delta T$  contribute to the total rms phase jitter which is a measure of the ranging accuracy. The system in Figure 2 measures the rms phase jitter as a function of  $C/N_O$  with the simulator settings as parameters. The Appendix explains how the computing counter is used to compute rms phase jitter.

# 5. RESULTS - Q-M/PSK HYBRID MODEM

In Figure 3 plots of BER vs  $C/N_0$  are given for performance of the data demodulator portion of the Q-M/PSK Hybrid Modem under various operational conditions. The curve on the left is the theoretical performance curve for a differentially encoded phaseshift keyed system (DECPSK). The next curve is a straight data curve with no mulitpath. This curve was established from laboratory data gathered after the ATS-6 technology tests. The curve is significant in that it agrees with earlier curves established for the operation of this modem (ref. 2), and in doing so, shows that no performance degradation has occurred with time. The next two curves of this graph represent the BER performance of the modem when operating in the voice-plus-data mode. These curves show that the performance of the modem in the voice-plus-data mode with phonetically balanced (PB) words as the voice input is better than the performance in the same mode with continuous speech as the voice input. When operating in the combined mode, the power is shared equally between the voice phase and data phase, but during pauses, the power is automatically pushed back to the data phase resulting in an improvement in BER. With PB words there are longer pauses between words (2.5 seconds) than with continuous talking, resulting in slightly improved performance.

The two right curves were established by using the modem in the voice-plus-data mode with PB words as the voice input and with the addition of multipath in a 10 Hz bandwidth. As is also true for the data only mode, the multipath causes a severe

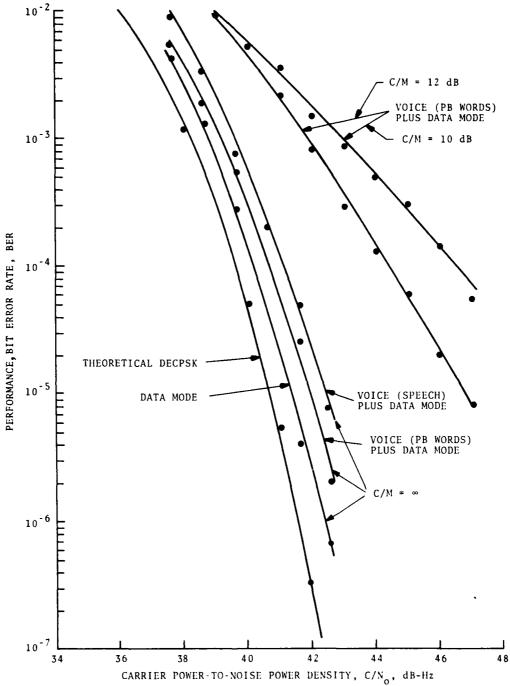


FIGURE 3. PERFORMANCE OF Q-M/PSK HYBRID MODEM IN THE VOICE PLUS DATA MODE

degradation in performance (ref. 2).

The following is a comparison of the various curves of Figure 3 at BER =  $10^{-5}$ :

20 2210 10 .	dB - Hz
Theoretical DECPSK	40.6
MODEM-Data only	41.3
MODEM-Voice (PB words) plus Data	41.9
MODEM-Voice (Speech) plus Data	42.4
MODEM-Voice (PB words) plus Data	
with Multipath (C/M=12 dB)	46.8
MODEM-Voice (PB words) plus Data	
with Multipath (C/M=10 dB)	>48

# 6. RESULTS - RANGING MODEM

Results of the laboratory tests of the ranging modem are shown in Figures 4 through 7. Figure 4 compares the four codes as a function of C/N<sub>O</sub> without multipath. The phase jitter resulting from the use of codes 1 and 3 is approximately an order of magnitude greater than that of codes 2 and 4. This is as expected since the inherent jitter in the loop is generally proportional to the clock tone period and inversely proportional to the clock tone power. Thus, code 4 has less rms phase jitter than code 2 because code 4 has more power in its clock tone. The same is true in the comparison of codes 1 and 3. Codes 2 and 4 require more bandwidth than codes 1 and 3 and so result in less jitter; thus, a tradeoff exists. To achieve greater accuracy, more bandwidth is required. To put rms phase jitter in terms of range accuracy:

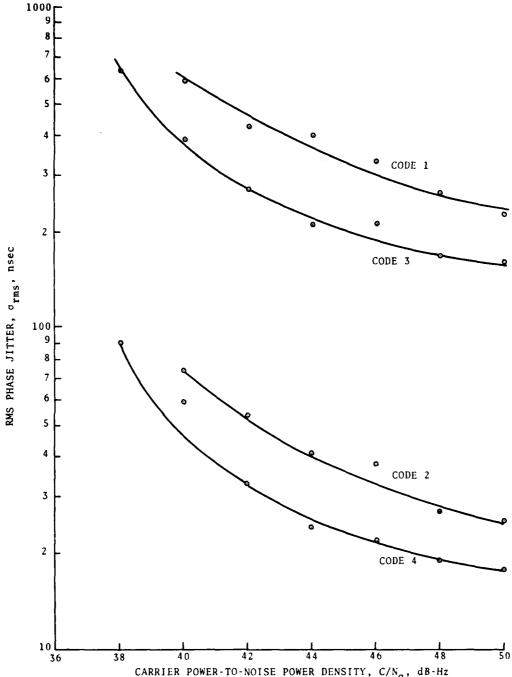
$$\Delta s = \overline{\nu} \Delta \tau \tag{1}$$

where  $\overline{v} = c = 3 \times 10^8 \text{ m/sec}$ 

Thus if  $\Delta \tau = 1$  nsec

then  $\Delta s = 3 \times 10^8 \times 10^{-9} = .3m = 1$  foot

Thus a 10-nsec rms jitter is approximately a 10-foot ranging error. Thus, if one desires a range accuracy of 30 feet at 42 dB-Hz, it would be necessary to use code 4 which requires about 156 kHz of bandwidth. However, if a range accuracy of only 300 feet is necessary at 42 dB-Hz, then code 3 can be used which requires approximately 19 kHz of bandwidth. These results are applicable for a no-multipath condition. A discussion of the results when multipath is added is given below.



CARRIER POWER-TO-NOISE POWER DENSITY, C/N<sub>o</sub>, dB-Hz
FIGURE 4. RANGING MODEM PERFORMANCE, RMS PHASE
JITTER, σ<sub>rms</sub>, VS CARRIER POWER-TONOISE POWER DENSITY, C/N<sub>o</sub>

Figures 5 and 6 again plot rms phase jitter as a function of C/N<sub>O</sub> but with varying amounts of multipath. As C/M varies from 11 dB to 2 dB, there is greater and greater degradation in system performance. However, even with C/M = 5 dB, the amount of degraation is less than a factor of two. The conclusion here is that the ranging error is far more a function of ranging code bandwidth than multipath. It should also be realized that the multipath bandwidth used was 1000 Hz. Further tests are necessary to establish range error for multipath bandwidths from 10 to 1000 Hz.

Figure 7 compares the performance of codes 3 and 4 to their respective theoretical performances. The theoretical performance curves were derived by combining the effects of loop noise: i.e.:

$$= \frac{1}{\Theta_0^2}$$

$$= 2 SNR_L \quad (ref. 5) \quad (2)$$

and quantization error. This equation is valid for  ${\rm SNR}_{\rm L}{>}10$ . As S/N decreases below this value higher order effects must be considered.

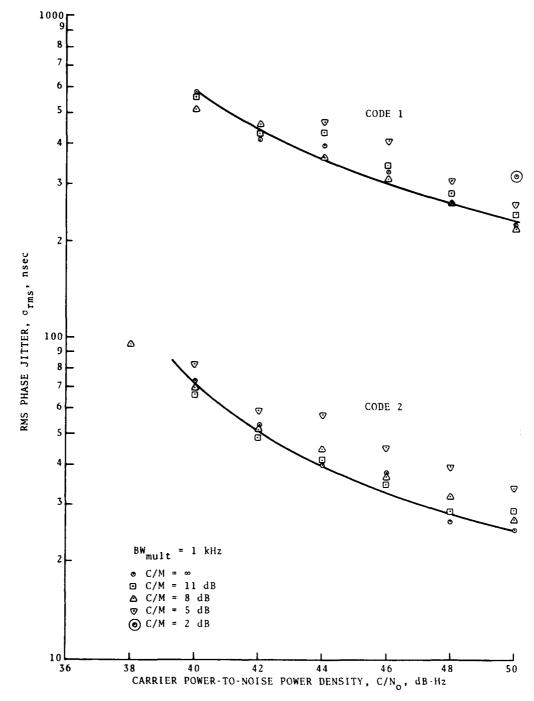


FIGURE 5. RANGING MODEM PERFORMANCE, RMS PHASE JITTER,  $\sigma_{\text{rms}}$ , VS CARRIER POWER-TO-NOISE POWER DENSITY, C/N  $_{\text{o}}$ , WITH MULTIPATH (CODES 1 AND 2)

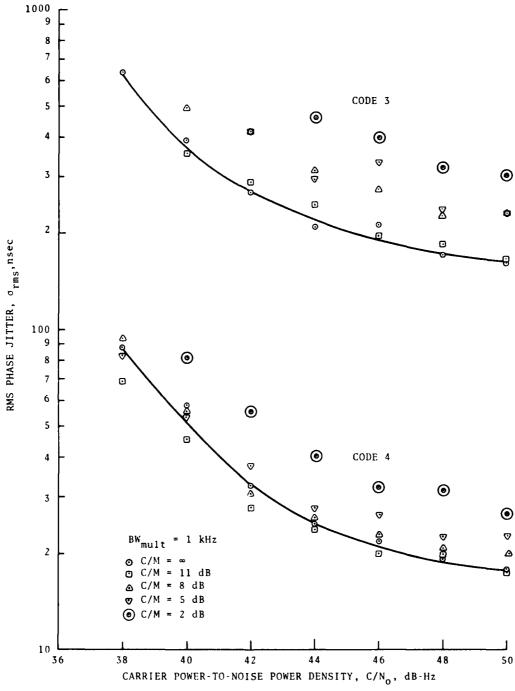


FIGURE 6. RANGING MODEM PERFORMANCE, RMS PHASE JITTER,  $\sigma_{\mbox{rms}}$ , VS CARRIER POWER-TO-NOISE POWER DENSITY, C/N  $_{\mbox{o}}$ , WITH MULTIPATH (CODES 3 AND 4)

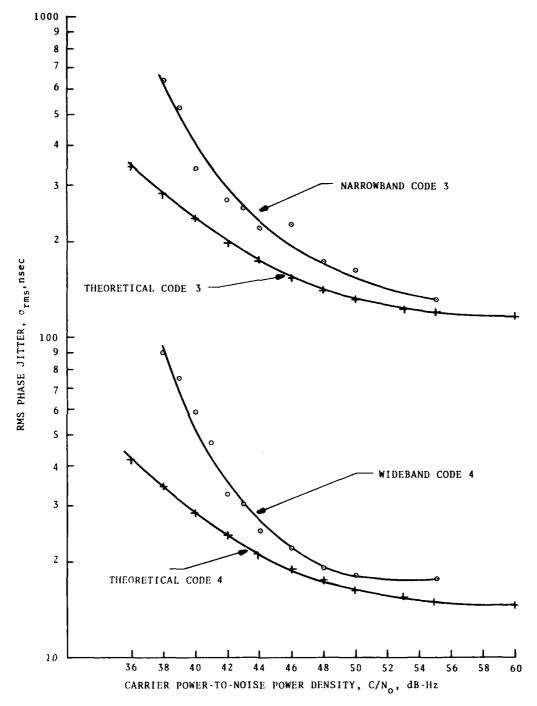


FIGURE 7. COMPARISON OF PERFORMANCE OF CODES 3 AND 4 TO THEORETICAL (RMS PHASE JITTER,  $\sigma_{\text{rms}}$ , VS CARRIER POWER-TO-NOISE POWER DENSITY,  $\text{C/N}_{\text{O}})$ 

## 7. CONCLUSIONS

The BEK performance of the Q-M/PSK Hybrid Modem is significantly degraded by multipath when operating in the voice plus data mode. For example, in the voice plus data mode, with PB words as the voice signal and a BER= $10^{-5}$ , a C/M of 12 dB results in a degradation of about 4.8 dB compared to a C/M= $\infty$ . Also, there is about a 0.6 dB degradation in BER performance in the combined mode when the voice signal is continuous speech as compared to a voice signal of PB words spaced 2.5 seconds apart.

The TSC ranging modem performs close to theoretical, at  $\text{C/N}_{0}$  greater than 42 dB-Hz. Below 42 dB-Hz the data representative of laboratory testing diverges from the theoretical. As mentioned earlier, this is a result of the failure to consider higher order effects in the theoretical curve calculations which become more significant in a noisier environment. The modem is marginally affected by multipath at the multipath fading bandwidth used. Further tests are necessary to establish system performance at other bandwidths.

#### **APPENDIX**

CALCULATION OF RMS PHASE JITTER

By definition: 
$$\sigma^2 = E\left\{ (\overline{x} - \eta)^2 \right\} = \int_{-\infty}^{\infty} (x - \eta)^2 f(x) dx$$
 (A-1)

where:

 $\eta$  = mean of the rv  $\bar{x}$ 

 $\sigma^2$  = variance of the rv  $\bar{x}$ 

 $\sigma$  = standard deviation of the rv  $\bar{x}$ 

if  $\bar{x}$  is of a discrete type, then:

$$\sigma^{2} = \sum_{\eta} (x_{\eta} - \eta)^{2} P(x = x\eta)$$
 (A-2)

note that:

$$\sigma^2 = E \left\{ (\bar{x} - \eta)^2 \right\} = E \left\{ x^2 - 2x\eta + \eta^2 \right\} = E \left\{ \bar{x}^2 \right\} - E^2 \left\{ \bar{x} \right\}$$
(A-3)

for a discrete rv:

$$E\left\{\bar{x}^2\right\} = 1/N \sum_{i=1}^{N} x_i^2 \qquad (A-4)$$

and 
$$E\left\{\bar{x}\right\} = 1/N \sum_{i=1}^{N} x_i$$
 (A-5)

Thus: 
$$\sigma = \left(\frac{1}{N}\sum_{i=1}^{N}x_{i}^{2} - (\frac{1}{N}\sum_{i=1}^{N}x_{i}^{2})^{2}\right)^{1/2}$$
 (A-6)

when  $x_i = \Delta t_i$ , we define  $\sigma = rms$  phase jitter

Thus: 
$$\sigma = \left[ \frac{N}{1/N} \sum_{i=1}^{N} (\Delta t_i)^2 - (1/N \sum_{i=1}^{N} \Delta t_i)^2 \right]^{1/2}$$
 (A-7)

The computing counter performs this computation at a selected rate of operation for a given N. The rate selected was .01/sec. and the selected N was N = 1000.

Thus, to compute  $\sigma$  requires 10 sec. However, because of noise,  $\sigma$  itself can really be considered a rv  $\bar{\sigma}$  with a  $\eta\bar{\sigma}$  and  $\sigma_{\bar{\sigma}}$  where  $\eta_{\bar{\sigma}}$  is the average of the rms phase jitter measurements and  $\sigma_{\bar{\sigma}}$  is the standard deviation of these measurements. Thus, the following quantities were computed:

$$n_{\tilde{\sigma}} = 1/N \sum_{n=j}^{N} \sigma j$$
 (A-8)

$$\varepsilon \sigma_{\bar{\sigma}} = \left(1/N \sum_{j=1}^{N} \sigma_{j}^{2} - (1/N \sum_{j=1}^{N} \sigma_{j})^{2}\right)_{(A-9)}^{1/2}$$

where N = 12, by selection.

The computation time required for this is approximately three minutes.

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